Evaluation of the effect of morphological traits on fish growth by comparison using ridge and ordinary least squares regression

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Abstract

Growth in fish is characterized by length and weight and studies that encompassing relationships among the fish length and weigh with some morphometric traits provide crucial information in the field of fish biology. High correlations (or multicollinearity) among the morphometric traits in fish morphology studies is a well-known phenomenon. If the relationship between growth and morphometry is modeled using the ordinary least squares estimator (OLS), the parameter estimates are likely to be too large in absolute value and possibly have the wrong sign due to the problem of multicollinearity. The ridge regression (RR) estimator has been proposed to avoid the adverse effects of multicollinearity among the regressors. In this study, therefore, multiple linear RR was used to model relationship fish length and fish weight to some predictive metric traits. Predictive traits were predorsal length, head length, post dorsal length, head height, and eye diameter. Data were derived from a total of 126 Capoeta damascina individuals sampled from the Euphrates, Türkiye. The ridge optimal k-parameter, producing acceptable variance influence factor (VIF<10), was determined as 0.021 for both of growth indices considering a combination of ridge trace and VIF trace plots and MSE and VIF values produced from an array of k values (0.0≤ k≤1.00). Although, small decreases were observed in the adjusted R-square values (ARS) obtained by RR compared to the ARS values obtained by OLS, a significant decrease in VIF values outweighs this drawback, indicating that the models are more stable. The methodological approach and findings in this study may contribute to filling a gap in the literature regarding the relationships between fish growth and morphology. Additionally, it could enhance better growth predictions for different fish species, aiding sustainable fisheries management and the selective cultivation of desired fish traits in aquaculture by improving the understanding of morphometric features.

Keywords: Fish morphometry, Multicollinearity, Variance influence factor, Capoeta damascina

1. Introduction

Fish morphometric traits are quantifiable characteristics related to the shape, size, and proportions of fish bodies. Morphometric studies encompassing these relationships provide crucial information in the field of fish biology, including species growth, feeding behavior, ecological strategies, niche partitioning, habitat utilization, and trophic structure. For instance, examining the linear relationships between total length, body height, and width reveals growth patterns, the height of the caudal peduncle influences swimming efficiency and overall fitness, while pectoral fin length and eye diameter reflect adaptations related to habitat, feeding, and locomotion. These morphometric analyses contribute significantly to our understanding of fish diversity and ecological dynamics (Clabaut et al., 2007; Mojekwu & Anumudu, 2015; Keppeler et al., 2020; Pino et al., 2021). In addition to these, morphometric features can also be used in aquaculture to distinguish between fish escaped from the rearing environment and wild fish (Arechavala-Lopez et al., 2012; Lenhardt et al., 2012).
Morphometric characteristics are taken into account in stock discrimination studies in fisheries science using multivariate statistical methods such as Principal Component Analysis, Cluster Analysis, and Linear Discriminant Analysis (Winans, 1987; Turan, 1999; Mojekwu & Anumudu, 2015). But, due to allometric growth that is an inherent characteristic of fishes which is a common problem with morphometric data is that all measurements are highly correlated with length, the morphometric traits are required to be transformed before analysis in stock discriminant studies. This high correlation between morphometric features is called multicollinearity. Due to this situation, the simple linear regression method is generally used to evaluate the relationships between fish growth and morphometric characteristics, except for stock discrimination studies (Kyrizti & Moutopoulos, 2018; Orbach et al., 2019; Yazıcı & Yazıcıoğlu, 2020).

In cases where there is multicollinearity; (i) The values of parameters estimated by the Ordinary Least Squares (OLS) method may differ significantly from their actual values, (ii) Regression coefficients are uncertain and the standard errors of these coefficients are infinite, (iii) Variance and covariances of the regression coefficients are tend to increase, (iv) R-square value of the model is high. However, none or very few of the independent variables may be significant according to the partial t-test, and (v) The direction of the relationships of the relevant independent variables with the dependent variable may contradict theoretical and experimental expectations. Although there are statistical approaches such as principal component analysis (PCA) and removing the variable having VIF higher than 10 are used to overcome this problem, the use of these approaches may lead to information loss. Another approach to overcome this problem is to use ridge regression. Ridge regression is a statistical method used to estimate coefficients in multiple regression models when independent variables are highly correlated without removing any variables from model (Albayrak, 2005; Karakaş, 2008; McDonald, 2009; Büyüküysal & Öz, 2016; Nandi & Saikia, 2015; Özkale & Altun, 2021).

Although ridge regression has been used for different purposes in fish related studies (Rikardsen & Johansen, 2003; Okamura et al., 2017; Niu et al., 2019; Martins et al., 2021; Liyandja et al., 2022; Gao et al., 2024), it seems that it has not been used to describe the relationship between fish morphology and fish growth to date. In essence, the fish material used in this study were considered for example purposes only to illustrate why and how ridge regression is used in modeling the relationship between fish growth and morphometry. Therefore, the main purpose of this study is to fill a gap in the literature on this subject by holistically estimating the relationship between total fish length and total weight characteristics and a group of morphometric features using the ridge regression method. Moreover, the methodological approach in the study can contribute to (i) sustainable fisheries management by making better growth predictions for different fish species and (ii) a better understanding.

2. Material and Methods

2.1. Fish material

The morphological features used in this study was belong to the *Capoeta damascina* species. The specimen measured in this study were provided from a research project that was supported by the Adiyaman University Scientific Research Projects (BAP) Coordination Unit, Project No. 2012/001, and was carried out with the legal consent of the Ministry of Food, Agriculture, and Livestock, General Directorate of Fisheries and Aquaculture (permission date 05.04.2012; permission number: 01515). The *C. damascina* species group occurs in the entire Levant, Mesopotamia, the Orontes, Iran, and the southern and eastern parts of Türkiye (Alp et al., 2005; Alp et al., 2013; Asadollah et al., 2017). A total of 126 *C. damascina* specimens caught from the Adiyaman region (in the middle Euphrates Basin) of Türkiye between April 2012 and December 2013 were caught in streams by using electroshock devices. Then, all length related measurements (mm) were made with a digital caliper with a precision of 0.01 mm, and the weight measurements were made with a digital scale with a precision of 0.01 g. Since this study was based on a modeling methodology approach, there was no need to provide further information about the fish and the area where they were sampled.

2.2. Metric traits

In the study total lengh (TL), total weight (TW), body height (BH), predorsal length (PR), head length (HL), post dorsal length (PD), head height (HH) and eye diameter (ED) metric features were taken into consideration.

2.3. Statistical framework

2.3.1. The general framework of the study

In the study total length (TL), total weight (TW), body height (BH), predorsal length (PR), head length (HL), post dorsal length (PD), head height (HH) and eye diameter (ED) metric features were taken into consideration. Irrespective of used domain, the biggest debate with the ridge estimator is the selection of the regularization parameter (k). Therefore, the proposed general framework of the study was given below in hierarchical format:

a. Calculation of some descriptive statistics of all variables

b. Calculation and visualization of the correlation matrix between variables: It will give us a preliminary idea about whether there is a multicollinearity problem.

c. Calculation of VIF values for predictor variables based on Ordinary Least Square (OLS): If there is a doubt determined in step “b”, VIF values, which are
indicative of multicollinearity, are calculated for each predictor variable. The presence of variables with VIF equal and greater than 10 quantitatively indicates the presence of multicollinearity.

d. To have an idea about the approximate optimum k-parameter visually using graphical methods (VIF Trace and Ridge Trace Plot) for a range of the Ridge k parameter.

e. Calculate the models’ minimum Mean Square Error (MSE), minimum generalized cross-validation (GCV) and adjusted R-squared values for a range of Ridge k parameters.

f. Calculate the VIF values of the predictor variables for a range of the ridge k-parameter.

g. Deciding on the optimum ridge k value by considering the smallest MSE, adj-R-square and VIF values in a balanced manner.

2.3.2. Models

In this study TL and TW were considered dependent and other metric traits were considered independent (regressor) variables. Regardless relationship between dependent and independent variables, to simplify the modelling dependent variables were modelled with multiple linear regression approach as following:

\[
TL = \alpha + \beta_1 PR + \beta_2 PD + \beta_3 HH + \beta_4 BH + \beta_5 HL + \beta_6 ED + \varepsilon
\]

\[
TW = \alpha + \beta_1 PR + \beta_2 PD + \beta_3 HH + \beta_4 BH + \beta_5 HL + \beta_6 ED + \varepsilon
\]

where, \(\alpha\), \(\beta\), and \(\varepsilon\) are intercept, regression coefficients, and error term, respectively.

2.3.3. Multicollinearity and variance inflation factor (VIF)

In regression analysis, some assumptions are taken into account when estimating the parameters that show the relationship between dependent and independent variables. In order to determine the relationship between variables with multiple regression analysis, the assumptions of the multiple linear regression model must be met. However, the assumptions of the multiple regression model are not valid in the analysis of every event. The assumptions in question are the assumptions about the error vector and the independent variable matrix. One of these assumptions that ensures that the standard errors of the estimates are smallest and therefore the parameter estimates are effective is that there is no relationship between the independent variables. This is called multicollinearity. As can be seen from the graphs above (Figure 1 and Figure 2), the existence of high correlations between the predictor variables is a sign of this situation. If there is a perfect correlation between the explanatory variables, that is, if the correlation coefficient for these variables is equal to 1, the parameters become undeterminable. It becomes impossible to find separate numerical values for each parameter, and in this case the least squares method cannot be used. On the other hand, if there is no correlation between the explanatory variables, that is, if the correlation coefficient for these variables is equal to 0, they are called orthogonal variables and do not pose a problem in estimating the coefficients. Multicollinearity may occur for various reasons (Katz, 2006; Adeboye et al., 2014; Daoud, 2017). However, considering the current study, the main reason for multicollinearity is that the variables of interest tend to change together over time.

One of the approaches used to detect multicollinearity is the variance inflation factor (VIF). The variance inflation factor is calculated to determine the degree of relationship of an independent variable with other independent variables. If there is no relationship between the dependent variable and the independent variables (\(R\)-square=0), the variance inflation factor is equal to 1 (1/(1- \(R\)-square)=1/(1-0)=1). If there is a perfect relationship between the dependent and independent variables (\(R\)-square=1), the variance inflation factor \([VIF=1/(1- \(R\)-square)=1/(1-1)]\) will be infinite. If \(R\)-square=90\%, the variance inflation factor is obtained as 10 [VIF=1/(1-0.90)=10]. In general, the following general rule applies for interpretation: if the variance inflation factor is equal to or greater than 10 (VIF>10), there is a multicollinearity problem (Thompson et al., 2017; Oke et al., 2019). In case of collinearity, it is necessary to eliminate it in order to reduce the standard error of coefficient estimates that determine the relationship between dependent and independent variables and to make more consistent estimates. One of the methods used in case of multicollinearity is Ridge regression.

2.3.4. Ridge regression

The solution technique of ridge regression (RR) is similar to the ordinary least squares (OLS) method. The RR method, is performed by adding a small and positive constant to the diagonal elements of the \((X^TX)\) matrix formed by the variables in standard form before calculating the regression coefficient estimates. Accordingly, the ridge regression solution is as follows;

\[
\hat{\beta}_{RR} = (X^TX + kI)^{-1}X^TY
\]

In the formula “I” and “k” represent the identity matrix and the ridge parameter, respectively. The difference of the RR method from OLS is the presence of the k-ridge parameter. Since the ridge solution for \(k=0\) is equivalent to the OLS solution, the ridge estimate can also be expressed as a linear transformation of the least squares estimate. In the RR method, the steps followed in the least squares method are repeated more than once. Among the parameter estimates calculated for each k that has a value between 0 and 1, those that meet the sought criterion are determined.

One of the main obstacles in using RR is in choosing an appropriate value of \(k\). The inventors of RR suggested
using a graphic which they called the ridge trace. This plot shows the RR coefficients as a function of \( k \). When viewing the ridge trace, the analyst picks a value for \( k \) for which the regression coefficients have stabilized. Often, the regression coefficients will vary widely for small values of \( k \) and then stabilize. Choose the smallest value of \( k \) possible (which introduces the smallest bias) after which the regression coefficients seem to remain constant. Note that increasing \( k \) will eventually drive the regression coefficients to zero.

2.4. Analysis tools

In the study, the packages “lmridge” (Ullah et al., 2018), “psych” (Revelle & Revelle, 2015), and “rattle” (Williams, 2011) in the R program were used.

lmridge contains functions related to fitting of the RR model and provides a simple way of obtaining the estimates of RR coefficients, testing of the ridge coefficients, and computation of different ridge related statistics, which prove helpful for selection of optimal biasing parameter \( k \). The four arguments of lmridge() function are described in Table 2 (Ullah et al., 2018). The syntax of default function is,

```
lmridge(formula, data, scaling = c("sc", "scaled", "centered"), k, ...)
```

Table 1. Description of lmridge() function arguments

<table>
<thead>
<tr>
<th>Argument</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formula</td>
<td>Symbolic representation for RR model of the form, response ~ predictors.</td>
</tr>
<tr>
<td>Data</td>
<td>Contains the variables that have to be used in RR model.</td>
</tr>
<tr>
<td>( k )</td>
<td>The biasing parameter, may be a scalar or vector. If a ( k ) value is not provided, ( k=0 ) will be used as the default value, i.e., the OLS results will be produced.</td>
</tr>
<tr>
<td>Scalling</td>
<td>The methods for scaling the predictors. The sc option uses the default scaling of the predictors in correlation form; the scaled option standardizes the predictors having zero mean and unit variance; and the centered option centers the predictors.</td>
</tr>
</tbody>
</table>

There are many scientific approaches to choosing the correct ridge parameter. The ultimate choice of \( k \) for a particular application involving linear explanatory variables still remains part art, part science (Forrester & Kalivas, 2004; Khalaf & Shukur, 2005; McDonald, 2009; Mami et al., 2021). In this study, in deciding on the \( k \) parameter, a series of \( k \) parameter (0.0\( \leq k\leq 1.0 \)) was considered into iterative manner in each of steps described under general framework of the study.

3. Results

3.1. Descriptive statistics and correlations

Some statistics (mean, standard deviation, coefficient of variation) of the metric features taken into account in the research are given in Table 1. Cross-correlations between variables, histogram distributions of the variables, and Correlation dendrogram are given in Figure 1 and Figure 2, respectively. In terms of variability (measured by Cv, %), TW had the highest (84.09%) as a nature of weight, others somehow were similar except ED (14.97%).

Table 2. Mean, standard deviation (Sd) and coefficient of variation (CV, %) values of the variables

<table>
<thead>
<tr>
<th>Metric trait</th>
<th>Mean</th>
<th>Sd</th>
<th>CV, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>ED</td>
<td>6.13</td>
<td>0.92</td>
<td>14.97</td>
</tr>
<tr>
<td>BH</td>
<td>34.96</td>
<td>8.82</td>
<td>25.22</td>
</tr>
<tr>
<td>HH</td>
<td>21.70</td>
<td>6.10</td>
<td>28.12</td>
</tr>
<tr>
<td>HL</td>
<td>32.48</td>
<td>8.07</td>
<td>24.86</td>
</tr>
<tr>
<td>PD</td>
<td>54.38</td>
<td>16.15</td>
<td>29.71</td>
</tr>
<tr>
<td>PR</td>
<td>68.61</td>
<td>17.04</td>
<td>24.84</td>
</tr>
<tr>
<td>TL</td>
<td>171.16</td>
<td>45.04</td>
<td>26.32</td>
</tr>
<tr>
<td>TW</td>
<td>58.58</td>
<td>49.26</td>
<td>84.09</td>
</tr>
</tbody>
</table>

As can be seen in Figure 1 and Figure 2, there are quite high correlations between the predictor variables (ED, BH, HH, HL, PD, and PR), and between predictors and dependent variables (TL and TW). High correlations observed between independent variables indicate a
potential multicollinearity problem. However, since simple correlation coefficients alone are not sufficient, it is not possible to be sure of the existence of a multicollinearity problem (Ölmez & Yaşanoğlu, 2023).

3.2. Ordinary Least Square (OLS) results and VIF values

Ridge solution for \( k=0 \) is equivalent to the ordinary least squares solution (OLS). In case \( k=0 \) for TL, the RR analysis resulted that \( \text{adj-} R^2=0.984 \), MSE=8626.853, and regression coefficients of PR, PD, BH, and HL were significant \((p<0.05)\), whereas HH and ED were not \((p>0.05)\). For TW, \( \text{adj-} R^2=0.886 \), MSE=127826.47, and only regression coefficients of BH and ED were significant \((p<0.05)\), in contrast other predictors were not \((p>0.05)\) (Figure 3, Figure 4).

VIF values of predictor variables of both TL and TW based on ordinary least squares (OLS) were calculated as: PR=60.84, PD=36.22, HH=46.02, BH=14.97, HL=50.27, and ED=4.50. As suspected above from correlation results, VIF values clearly show that there was multicollinearity among the predictor variables for both of TL and TW (Figure 3 and Figure 4).

3.3. Searching for optimal “k” value

3.3.1. Graphical Searching

In graphical searching, the ridge trace plots and VIF trace plot for a range of \( k \) parameter \((0.0 \leq k \leq 1.0)\) for TL and TW estimations were given in Figure 5, respectively. Ridge Trace Plot shows the effect of different \( k \)-ridge values on the regression coefficient of each predictor variable for dependent variable (TL and TW). VIF Trace plot shows the effect of different \( k \)-ridge values on the VIF values of each predictor variable for the dependent variable (TL and TW). As can be seen from the “Ridge Trace Plot” graphs for TL and TW, as the \( k \) values increase, the “Ridge” beta coefficients get closer to each other. On the graph, it is reported that the smallest MSE (7131.745) value for TL was reached at \( k=0.003 \). For TW, the lowest MSE value (59893.122) was reached at \( k=0.016 \).

VIF Trace plot shows the decrease in VIF values of predictor variables despite increasing \( k \) values according to generalized cross-validation or GCV criterion (Golub et al., 1979). The minimum GCV value for TL was reached at \( k=0.003 \), and the minimum GCV value for TW was reached at \( k=0.018 \).

3.3.2. Searching of the best \( k \) by minimum MSE

According to the minimum MSE approach, the ridge \( k \) value that produces the least MSE value (7131.745) for TL prediction is 0.003 and the \( \text{adj-} R^2 \) value of the model is 0.9825 (Figure 6), while for TW prediction these values are found to be 59893.12, 0.016 and 0.8722, respectively (Figure 7).
3.3.3. Searching of the best $k$ by acceptable VIF

Figure 8 gives the VIF values of each estimator variable for different $k$-ridge parameter values for TL and TW modelling. In contrast to the $k$ values increased, VIF values were values decreased. The first $k$ value where the VIF values of the predictor variables were all calculated below 10 was found to be 0.021.

3.3.4. Final decision on the optimum $k$ parameter

So far, to decide the optimum $k$ parameter, we have used the graphical method, calculating MCE, VIF and $R^2$-squared values. In graphical methods, we considered Ridge Trace Plot and VIF Trace Plot. In the Ridge Trace Plot, the $k$ value that produces the lowest MCE is also given on the graph. For TL, $k=0.003$ and MCE=7131.745, for TW, $k=0.016$ and MCE=59893.122 values were produced.

In VIF Trace Plot, the $k$ value that produces the minimum GCV value is also given on the graph. While the $k$ value that produced the lowest GCV for TL (0.325) was 0.003, the $k$ value that produced the minimum GCV for TW (4.757) was found to be 0.018.

When we searched for the minimum MCE values for different $k$ values one by one, it was seen that the $k$ value that produced the minimum MCE value was the same as the Rigg Trace Plot for TL and TW predictions. However, the first $k$ value where each predictor variable in the model met the condition of a VIF value less than 10 was found to be $k=0.021$ for both predictions. In this case, it will be necessary to choose between the values of $k=0.003$ and $k=0.0214$ for TL and the values of $k=0.016$, $k=0.018$ and $k=0.0214$ for TW.

Although the lowest MCE was obtained when $k=0.003$ was selected for TL (7131.745), the VIF value of no predictor variable, except for the ED predictor variable (4.29), did not fall below the acceptable threshold value (VIF<10). On the other hand, if $k=0.021$ is selected, the MCE will increase slightly (17028.546) and the adj-$R$-squared value will decrease slightly (from 0.9825 to 0.9726), and the VIF values of the predictor variables will be reduced to acceptable limits (VIF<10) (Figure 9).

Figure 9. VIF (VIF) values and some statistics ($rstats1$) for $k\{0.003, 0.021\}$ values for TL estimation
If \( k = 0.016 \) and \( k = 0.018 \) are selected for TW, lower MSE (59893.12 and 60047.47) and higher adj-R-square (0.8722 and 0.8688) values can be obtained. However, when \( k = 0.016 \), the VIF values of the PR, PD, HH and HL predictor variables are greater than the acceptable threshold value (VIF>10). On the other hand, if \( k = 0.016 \), the VIF values of the predictor variables drop to an acceptable level (VIF<10) (Figure 10).

Considering all the above findings, larger \( k \) values have the potential to both bring the regression coefficients closer to zero and produce lower adj-R-squared values. In addition, lower \( k \) values have the potential to produce higher VIF values and some potentially important predictor variables to be insignificant. Therefore, it was decided to take the optimum ridge \( k \) value as 0.021 for TL and TW predictions.

### 3.4. Ridge regression analysis results

The ridge regression result for TL and TW estimates based on \( k = 0.021 \) value were given in Figure 11 and Figure 12, respectively. When looking at the ridge regression results in relation to TL, except for the HH and ED variables, the coefficients for the other predictor variables were found to be non-significant (\( p \geq 0.05 \)). The effects of predictor variables on TL were determined as HL>PR>PD>BH, from largest to smallest. The regression equation was estimated as follows:

\[
TL = -0.96 + 0.79PR + 0.70PD + 0.45BH + 1.67HL \\
(Adj-R-square= 0.9726)
\]

According to the ridge regression results in relation to TW, except for PD and ED variables, regression coefficients for other predictor variables were found to be significant (\( p < 0.05 \)). The effects of predictor variables on TW were determined as BH>HL>HH>PR from largest to smallest. The regression equation for the relationship was estimated as follows:

\[
TW = -143.99 + 0.76PR + 2.43HH + 2.91BH + 1.95HL \\
(Adj-R-square= 0.868)
\]

### 4. Discussion

For over three decades, ridge regression has proven to be a valuable tool for applied statisticians and should be routinely investigated in the context of collinear multiple regression (McDonald, 2009). As a regularized regression method, ridge regression reduces the chances of overestimating coefficients, and makes predictive and explanatory models more informative and interpretable, especially for datasets with strong collinearity. The parameter estimation also has less variance than that generated by iterating variables and is therefore more stable (Niu et al., 2019). Ridge regression adjusts the least squares estimator by reducing the impact of multicollinearity on parameter estimates. The resulting equation consists of least squares and estimates that are not biased but are also more stable; This is an important stability for models to be used in prediction (Rikardsen & Johansen, 2003). Sahin et al (2018) tried to estimate the impact of some traits on the albumen index value that is one of the interior egg quality traits using the Least Squares (LS), Ridge Regression (RR) and Principal Components Regression (PCR). Due to the multicollinearity between independent variables, the standard errors and VIF values of the partial regression coefficients in the LS method were found to be quite high. They concluded that the use of RR and PCR analysis methods could be more accurate instead of the LS method under multicollinearity problem.

OLS (\( k = 0 \) case) and RR (\( k = 0.021 \)) regression analysis results for TL and TW predictions are given together in Figure 13 and Figure 14. When looking at the OLS and RR regression results in Figure 13, it is seen that the regression coefficients are different and the importance levels of the BH and PD predictor variables in RR also increase. However, as expected in RR (Ölmez & Yağanoğlu, 2023), there was a slight decrease in the adj-R-square value (from 0.9848 to 0.97).
When OLS and RR regression results for TW are compared (Figure 14), it is seen that the regression coefficients of the variables have changed. It was determined that the significance level of the HL variable increased in RR, and although the regression coefficient of the ED variable was significant in OLS ($p < 0.05$), it was found to be insignificant in RR ($p > 0.05$). As in the TL estimation, the adj-R-square value decreases slightly when TW is modeled with RR (from 0.87220 to 0.8685).

Among some measure on collinearity, it was pointed that the VIF (individual measure) had a better performance on the fitted model on the morphology of *C. macropomum*. Also, the presence of collinearity indicates that a substantial part of the information in one or more of these covariates is redundant (Rivas Villegas et al., 2024). Therefore, the higher adj-R-square values in OLS compared to RR could be expected. As supported in our case, the relationships of both TL and TW with the predictor variables, a small decrease was observed in the adjusted R-square values obtained by ridge estimation compared to the adjusted R-square values obtained by the OLS estimation method. However, a significant decrease in VIF values outweighs this drawback, indicating that the models are more stable. In the study, estimates were made using all fish caught, without distinguishing between male and female individuals. In a purely ecological study, this situation can be considered a limiting situation in terms of the generalizability of the study results. However, this can be ignored since the current study was not an ecological study and the focus of the study was how ridge regression was used methodologically in evaluating the fish morphology-growth relationship.

5. Conclusion

Fish growth occurs in length and weight. Although it is known that there is a relationship between morphometric characteristics fish growth, modeling is limited to simple linear regression due to the high level of multicollinearity among predictive variables. One of the methods used in cases of multicollinearity is the ridge regression method. However, in this approach, determining the optimum ridge regression parameter (penalization term, k or lambda) can often be speculative and difficult. In the study, using multiple linear models for total length (TL) and total weight (TW) ($Y = \alpha + \beta_1X_1 + \ldots + \beta_nX_n + \varepsilon$), the holistic effect of metric features on these two features was estimated using ridge regression. Measurements of a total of 126 *Capoeta damascina* individuals sampled from the Euphrates River (around Adıyaman province, Türkiye) were used as the data set. Predorsal length (PR), head length (HL), post dorsal length (PD), head height (HH) and eye diameter (ED) predictive metric features were considered. The ridge optimum $k$-parameter, which produces acceptable values (VIF<10) of the variance inflation factor (VIF) of the predictor variables, which are indicators of multicollinearity in the modeling, was determined as $k=0.021$ in both models. With the Ridge estimation method, in the relationship between total height (TL) and predictive metric traits ($TL= -0.96 + 0.79PR + 0.70PD + 0.45BH + 2.91HL$), except for ED and HH, other predictive variables were statistically significant ($p<0.05$). For example, eye diameter and post dorsal length (PD), it was determined that the effects of other metric traits on total weight were significant ($p<0.05$), and the relationship between total length and important predictive features was estimated as $TW= -143.99 + 0.76PR + 2.43HH + 2.91BH + 1.95HL$.

Among some measure on collinearity, it was pointed that the VIF (individual measure) had a better performance on the fitted model on the morphology of *C. macropomum*. Also, the presence of collinearity indicates that a substantial part of the information in one or more of these covariates is redundant (Rivas Villegas et al., 2024). Therefore, the higher adj-R-square values in OLS compared to RR could be expected. As supported in our case, the relationships of both TL and TW with the predictor variables, a small decrease was observed in the adjusted R-square values obtained by ridge estimation compared to the adjusted R-square values obtained by the OLS estimation method. However, a significant decrease in VIF values outweighs this drawback, indicating that the models are more stable. In the study, estimates were made using all fish caught, without distinguishing between male and female individuals. In a purely ecological study, this situation can be considered a limiting situation in terms of the generalizability of the study results. However, this can be ignored since the current study was not an ecological study and the focus of the study was how ridge regression was used methodologically in evaluating the fish morphology-growth relationship.

5. Conclusion

Fish growth occurs in length and weight. Although it is known that there is a relationship between morphometric characteristics fish growth, modeling is limited to simple linear regression due to the high level of multicollinearity among predictive variables. One of the methods used in cases of multicollinearity is the ridge regression method. However, in this approach, determining the optimum ridge regression parameter (penalization term, k or lambda) can often be speculative and difficult. In the study, using multiple linear models for total length (TL) and total weight (TW) ($Y = \alpha + \beta_1X_1 + \ldots + \beta_nX_n + \varepsilon$), the holistic effect of metric features on these two features was estimated using ridge regression. Measurements of a total of 126 *Capoeta damascina* individuals sampled from the Euphrates River (around Adıyaman province, Türkiye) were used as the data set. Predorsal length (PR), head length (HL), post dorsal length (PD), head height (HH) and eye diameter (ED) predictive metric features were considered. The ridge optimum $k$-parameter, which produces acceptable values (VIF<10) of the variance inflation factor (VIF) of the predictor variables, which are indicators of multicollinearity in the modeling, was determined as $k=0.021$ in both models. With the Ridge estimation method, in the relationship between total height (TL) and predictive metric traits ($TL= -0.96 + 0.79PR + 0.70PD + 0.45BH + 2.91HL$), except for ED and HH, other predictive variables were statistically significant ($p<0.05$) (Adj-R-square=0.97). Except for eye diameter and post dorsal length (PD), it was determined that the effects of other metric traits on total weight were significant ($p<0.05$), and the relationship between total length and important predictive features was estimated as $TW= -143.99 + 0.76PR + 2.43HH + 2.91BH + 1.95HL$.
(Adj-R-square=0.87). The methodological approach and findings in this study may contribute to filling a gap in the literature regarding the relationships between fish growth and morphology. Additionally, it could enhance better growth predictions for different fish species, aiding sustainable fisheries management and the selective cultivation of desired fish traits in aquaculture by improving the understanding of morphometric features.

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Conflict of Interest
The authors declare that they have no known competing financial or non-financial, professional, or personal conflicts that could have appeared to influence the work reported in this paper.

Ethical Statement
The paper is not currently being considered for publication elsewhere, and it reflects the authors' own research and analysis in a truthful and complete manner.

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